Linear Programming Models II

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Example: Flair Furniture Co.

Two products: Chairs and Tables

Decision: How many of each to make this month?

Objective: Maximize profit

Flair Furniture Co. Data

	Tables (per table)	Chairs (per chair)	Hours Available
Profit Contribution	$7	$5	
Carpentry	3 hrs	4 hrs	2400
Painting	2 hrs	1 hr	1000

Other Limitations:

- Make no more than 450 chairs
- Make at least 100 tables

Model Summary

Max 7T + 5C (profit)

Subject to the constraints:

3T + 4C ≤ 2400 (carpentry hrs)

2T + 1C ≤ 1000 (painting hrs)

C ≤ 450 (max # chairs)

T ≥ 100 (min # tables)

T, C ≥ 0 (nonnegativity)

Graphical Solution

- Graphing an LP model helps provide insight into LP models and their solutions.
- While this can only be done in two dimensions, the same properties apply to all LP models and solutions.

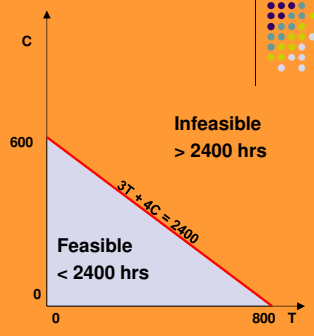
Carpentry Constraint Line

3T + 4C = 2400

Intercepts

(T = 0, C = 600)

(T = 800, C = 0)



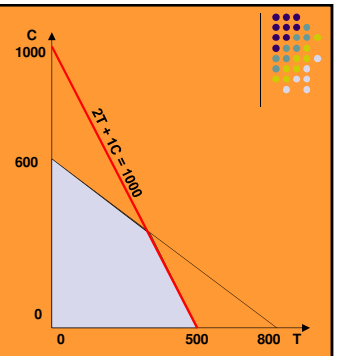
Painting
Constraint Line

2T + 1C = 1000

Intercepts

(T = 0, C = 1000)

(T = 500, C = 0)

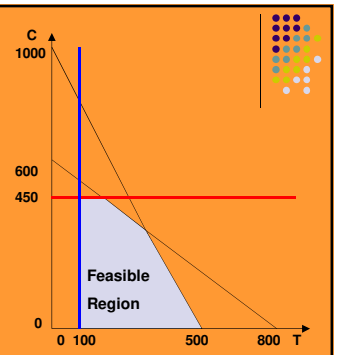


Max Chair Line

C = 450

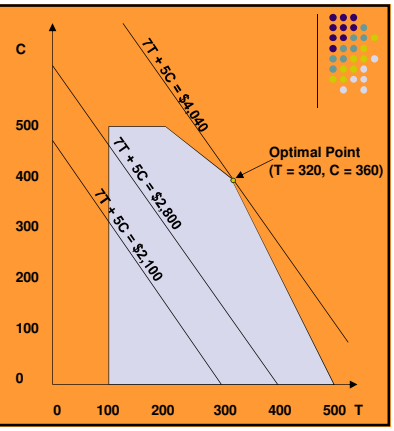
Min Table Line

T = 100



Objective
Function Line

7T + 5C = Profit



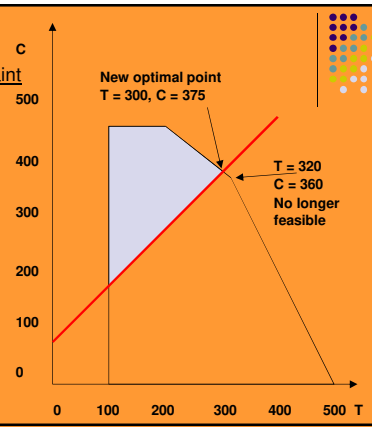
Additional Constraint

Need at least 75 more chairs than tables

$C \geq T + 75$

Or

$C - T \geq 75$



LP Characteristics

- **Feasible Region**: The set of points that satisfies all constraints
- **Corner Point Property**: An optimal solution must lie at one or more corner points
- **Optimal Solution**: The corner point with the best objective function value is optimal

Special Situation in LP

1. **Redundant Constraints** - do not affect the feasible region

   Example: $x \leq 10$

   $x \leq 12$

   The second constraint is redundant because it is *less* restrictive.

Special Situation in LP

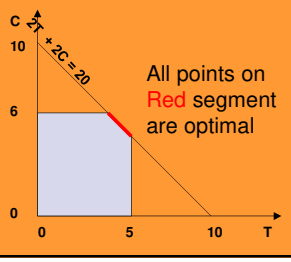
2. **Infeasibility** – when no feasible solution exists (there is no feasible region)

   Example: $x \leq 10$
   $x \geq 15$

Special Situation in LP

3. **Alternate Optimal Solutions** – when there is more than one optimal solution

Max 2T + 2C
Subject to:
$T + C \leq 10$
$T \leq 5$
$C \leq 6$
$T, C \geq 0$



All points on Red segment are optimal

Special Situation in LP

4. **Unbounded Solutions** – when nothing prevents the solution from becoming infinitely large

Max 2T + 2C
Subject to:
$2T + 3C \geq 6$
$T, C \geq 0$