Linear Programming Models

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Linear Programming: An Overview

- Objectives of organizations decisions frequently involve **maximizing profit/surplus** or **minimizing costs.**
- Linear programming uses **linear algebraic relationships** to represent a firm's decisions, given a business **objective**, and resource **constraints**.

Steps in Developing a Linear Programming (LP) Model

1) Formulation
2) Solution
3) Interpretation and Sensitivity Analysis

Properties of LP Models

1) Seek to minimize or maximize
2) Include “constraints” or limitations
3) There must be alternatives available
4) All equations are linear

Example LP Model Formulation: The Product Mix Problem

Decision: How much to make of ≥ 2 products?

Objective: Maximize profit

Constraints: Limited resources

Basic Assumptions of LP

- We assume conditions of *certainty* exist and numbers in the objective and constraints are known with certainty and do not change during the period being studied
- We assume *proportionality* exists in the objective and constraints
- We assume *additivity* in that the total of all activities equals the sum of the individual activities
- We assume *divisibility* in that solutions need not be whole numbers
- All answers or variables are *nonnegative*

Example: Flair Furniture Co.

Two products: Chairs and Tables

Decision: How many of each to make this month?

Objective: Maximize profit

Flair Furniture Co. Data

	Tables (per table)	Chairs (per chair)	Hours Available
Profit Contribution	$7	$5	
Carpentry	3 hrs	4 hrs	2400
Painting	2 hrs	1 hr	1000

Other Limitations:

- Make no more than 450 chairs
- Make at least 100 tables

Decision Variables:

T = Num. of tables to make

C = Num. of chairs to make

Objective Function: Maximize Profit

Maximize $7 T + $5 C

Constraints:

- Have 2400 hours of carpentry time available

  $3T + 4C \leq 2400$ (hours)

- Have 1000 hours of painting time available

  $2T + 1C \leq 1000$ (hours)

More Constraints:

- Make no more than 450 chairs

  $C \leq 450$ (num. chairs)

- Make at least 100 tables

  $T \geq 100$ (num. tables)

Nonnegativity:

Cannot make a negative number of chairs or tables

$T \geq 0$

$C \geq 0$

Model Summary

Max $7T + 5C$ (profit)

Subject to the constraints:

$3T + 4C \leq 2400$ (carpentry hrs)

$2T + 1C \leq 1000$ (painting hrs)

$C \leq 450$ (max # chairs)

$T \geq 100$ (min # tables)

$T, C \geq 0$ (nonnegativity)