4. Framework for Equitable Emission Reduction Apportionment

The principle of fairness demands that the global co-operation for mitigation evolves an entitlement based strategy of emission reduction apportionment by advancing the approach of UNFCCC in the form of '*common but differentiated responsibilities and respective capabilities and the social and economic conditions*'. It is undisputed that carbon emissions are in the nature of public "bad" produced by means of consumption activities in all countries. It is equally undisputed that every country and every individual is entitled for a certain quantum of energy for achieving growth and development. While the consensus regarding the basis of this entitlement needs to be arrived at, it may be assumed that the 1990 level average per capita emissions of the world would perhaps be the nearest to any such consensual basis. This assumption, on the aggregate, enables quantification of the energy entitlement for a country or region.

Aslam (2002) examines the merits and demerits of the per capita entitlement approach that leads to the following conclusions:

Merits

- Simplicity of concept
- Strong ethical basis
- Flexibility to accommodate changing scientific evidence
- Enhancement of efficiency of global trading
- Offer of incentives for developing-country participation
- Consistency with the major guiding principles of the UNFCCC
- Amalgamates well with the Kyoto architecture

Demerits

- Limited global acceptability
- Limited flexibility for accommodating varying country circumstances
- Linkage with trading essential for success
- Associated issues of hot air and obligation costs

In order to improve acceptability, he suggests some 'soft-landing scenarios' in the transition phase while maintaining that the ethical foundation of the per capita approach is likely to shape long term approaches.

An approach for such 'soft-landing scenarios' has been considered in Philibert et al. (2001) where five types of targets, namely, fixed, binding targets; dynamic targets; non-binding targets; sectoral targets; policies and measures, have been explored for non-Annex I countries. These are measures which can be voluntarily adopted by developing countries. However, these measures need to be associated with binding emission reduction commitments from Annex I countries.

An alternative approach, which has been adopted in this paper, is to arrive at per capita entitlement by computing the per capita emissions at the future target year, for which mitigation commitments are being implemented. A normative common per capita entitlement for all parties at a future target year would be appropriate in the context of emission reduction through convergence[19].

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[19] See also Dresner (2005) p.58

Dual Principle Approach to Apportionment

Differentiation of mitigation responsibility needs to be defined on a time and quantum scale taking into account the energy entitlements and also the cumulative emissions. The crucial factors in this context will vary from country to country depending upon the course of development. Bolin et al. (2001) argue that even with major and early control of CO_2 emissions by the developed world, the developing world also would need to control its emissions within decades. Therefore, formulation of a differentiated responsibility matrix, taking into account all the relevant factors is very important to bring in the equity perspective in the global emissions debate considering the position of the developing countries and transition economies.

Though a multi-criteria approach has been suggested in the literature to deal with emission apportionment, Cazorla (2000) points out that it is unlikely that a majority of nations would accept a multiple-criteria approach simply because it includes a lot of equity principles. Consensus will be difficult on the criteria and weights. Therefore it is considered that it would be more fruitful to focus on a few relevant and effective universal principles of fairness and equity. Krupnick et al. (2009) based on a multi-country survey of sharing the load of climate change mitigation reports that while 92% of Swedes and 71% of Americans are willing to pay for climate change mitigation efforts to the extent of 2–3% of the per-capita income, they prefer a *current emissions* principle (countries with high emission levels today would pay a larger share than countries with low emissions today) for dividing global mitigation costs among countries. The candidate principles for the survey were:

distributing the costs among countries by levels of *current emissions, historical emissions, income,* and *emissions per capita.*

While there are various candidate principles competing for legitimacy surveyed in the last chapter, a combination of 'polluter pays principle' as well as the 'ability to mitigate' have been selected to provide long term guidance in carbon dioxide mitigation, based on their universality and acceptance. A dynamic approach providing for continuous evaluation and assignment of differentiated responsibilities based on these considerations would be the most potent antidote to the emission build up. GDP or income has both forward and backward logical linkages to be the most natural ally of carbon mitigation, in as much as GDP or income is causally correlated with emissions while at the same time representing the ability for its mitigation. Thus it would be logical to generate the differentiated responsibility function as a linear combination of these two factors, namely, cumulative excess emissions and GDP.

It may be argued in this context that per capita GDP is a better measure of mitigation capacity than national GDP. Another possible candidate could be the available GDP (national GDP less population based entitlement) arrived at on the basis of the same principle of computation of excess emissions. However, in so far as population is also a driving factor of emissions (Masters, 1995, p.389), population based discounting has not been allowed on the mitigation capacity and the corresponding mitigation responsibility in the present study, though emission entitlements have been considered in computing excess emissions.

Methodological Framework

At a global level, the cumulative excess emissions are computed from a base year which in linear combination with the GDP provides the differentiated responsibility index for mitigation. The emission reduction required at each year at the global level may be computed from this function to achieve the target emission reduction at the final year of mitigation as well as the total quantum of emissions permissible during this period, provided the mitigation coefficients of this function are evaluated. The mitigation coefficients provide a bench mark for apportionment of the targets among various countries/regions also. In order to evaluate these coefficients, we proceed in an iterative manner by initially assuming feasible and flexible mitigation trajectories.

The possible mitigation trajectories which may be considered in this context are the constant pace mitigation and the parabolic mitigation (Socolow et al., 2007). There are other widely known stabilization trajectories also (Wigley et al., 1996). O'Neill (2004) suggests that the concentration trajectories that yield the same final concentration should consider the sensitivity to geophysical and ecological systems and not merely the path-dependent mitigation costs, as the likelihood of dangerous impacts increases under trajectories that delay emissions reduction or overshoot the final concentration (as in Elzen, 2007). This leads to issues of trajectory optimization, which we do not propose to examine in detail here.

Constant pace mitigation is not suitable for the current approach as it has only one variable parameter whereas the responsibility function is bivariate. It has been found that the cumulative gamma mitigation function for emission reduction mirrors the mitigation effort and its impact appropriately and offers sufficient flexibility for implementing mitigation trajectories for countries with diverse emission and income profiles. Alternatively, parabolic mitigation approach can also be employed. In fact, it has been found that the apportionment is more or less independent of the mitigation trajectory after fine tuning the trajectory in the feasibility region, which provides flexibility in optimizing the trajectories at policy planning levels.

Mathematical Formulation of the framework

Climate change modeling

Climate phenomena are complex interactions among many nonlinear variables and therefore climate modeling is quite complex. However, worthwhile results for macro variables can be obtained by means of the so called zero dimensional models which takes a macro-system perspective. There are the complex general circulation models which use three dimensional differential equations for energy and fluid flow and the interactions among them, which are integrated in time and space to arrive at time or space variations. But for many practical purposes, simple zero-dimensional models give useful results for policy planning and understanding. The following are some of the simple results which can be used for modeling (Masters, 2007):

$$Q_{rad}(W/m^2) = 1.83T_s(°C) + 209 \quad \text{(i)}$$

where Q_{rad} = Energy radiated from the top of the troposphere

T_s = Surface temperature

Climate sensitivity parameter $\lambda = 1/\left(\frac{\partial Qrad}{\partial Ts} - \frac{\partial Qabs}{\partial Ts}\right)$

$$\approx 0.55\ °C/\ W/m^2 \qquad \text{(ii)}$$

Q_{abs} = Energy absorbed at the top of the troposphere

Change in temperature due to a given radiative forcing (ΔT_s)

$$= \lambda \Delta F \qquad \text{(iii)}$$

where ΔF = Radiative forcing in W/m^2

Radiative forcing due to principal greenhouse gases may be estimated by the following equation:

$\Delta F\ (W/m^2)$ =

$$6.3 \times \ln\frac{[CO_2]}{[CO_2]_0} + 0.031(\sqrt{CH_4} - \sqrt{(CH_4)_0}) + 0.133(\sqrt{N_2O} - \sqrt{(N_2O)_0}) \qquad \text{(iv)}$$
$$+ 0.22[(CFC_{11}) - (CFC_{11})_0] + 0.28[(CFC_{12}) - (CFC_{12})_0]$$

where concentrations are in ppb

If the carbon content of the global atmosphere at time t (in years) is denoted by C(t), which is in units of billions of metric tons of carbon (GtC), and the global annual rate of CO2 emission to the atmosphere is denoted by E(t) in units of GtC/year, we have the following empirical relations (Socolow et al., 2007):

$$\frac{dC(t)}{dt} = k\mathrm{E}(t) \qquad \text{(v)}$$

where k (air-bourne fraction) ≈ 0.5

$$E_{stab} = \frac{C_{stab} - 600}{200} \qquad \text{(vi)}$$

where C_{stab} = Stabilization value of C(t) in GtC

E_{stab} = value of E(t) associated with C(t) stabilized at Cstab in GtC/year

Cumulative Gamma Probability Density Function (Pdf) Mitigation

If $\boldsymbol{E_m(t)}$ represents the emissions at any time in the Business As Usual(BAU) scenario and $\boldsymbol{E(t)}$ represents the emissions post-mitigation, then $\boldsymbol{E_m(t)}$ - $\boldsymbol{E(t)}$ is assumed to follow cumulative Gamma probability density function(pdf) so that the rate of emission reduction follows gamma pdf. Gamma distributions are used in a number of applications such as reliability assessment, queuing theory, computer evaluations, biological studies etc.

For $\alpha > 0$, the Gamma function, which extends the domain of factorials to non-integers is defined as follows:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \tag{1}$$

The Gamma function has the following useful properties:

- For any $\alpha > 1$, $\Gamma(\alpha) = (\alpha - 1)\ \Gamma(\alpha - 1)$
- For any positive integer, n, $\Gamma(n) = (n-1)!$

The gamma distribution, a semi-infinitely bounded unimodal distribution, represents the sum of n exponentially distributed random variables. Typically, the gamma distribution is defined in terms of a scale parameter $\boldsymbol{\alpha}$ and shape parameter $\boldsymbol{\beta}$, both of which may be non-integers. When used to describe the sum of a series of exponentially distributed variables, the shape factor represents the number of variables and the scale factor is the mean of the exponential distribution. Exponential, Chi-squared and Erlang distributions are special cases of the gamma distribution.

Gamma distribution can be used in applications based on intervals between events since it is the sum of one or more exponentially distributed variables. Eg: Queuing models. Due to its moderately skewed profile, it can be used as a model in a range of disciplines, including climatology where it is used for rainfall modelling. The profile of cumulative gamma function is suitable for modelling $\boldsymbol{E_m(t)} - \boldsymbol{E(t)}$.

The Gamma distribution pdf is defined (for α > 0; β > 0) using the Gamma function, as follows:

$$f(x;\alpha,\beta)=\begin{cases}\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta} & x\geq 0\\ 0 & otherwise\end{cases} \tag{2}$$

Mean and Variance of gamma pdf are given by:

$$E(X)=\mu=\alpha\beta$$
$$V(X)=\sigma^2=\alpha\beta^2$$

To find the cumulative gamma distribution function, we define the standard Gamma function as **f(x; α, 1)** so that the cumulative gamma distribution of the standard gamma pdf is given by:

$$F(x;\alpha)=\int_0^x \frac{y^{\alpha-1}e^{-y}}{\Gamma(\alpha)}dy \quad \mathrm{X}>0 \tag{3}$$

The above cumulative standard gamma function is known as the incomplete gamma function[20]. The cumulative gamma distribution of non-standard gamma distribution pdf can now be evaluated by:

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[20] The ordinary gamma function, obtained by integrating across the entire positive real line, is called the *complete gamma function*

$$F(x;\alpha,\beta) = F\left(\frac{x}{\beta};\alpha\right) \qquad (4)$$

where $F(\cdot\,;\alpha)$ is the incomplete gamma function

$$\boldsymbol{E(t) = E_m(t) - F}(t;\ \alpha, \beta) \qquad (5)$$

where $\boldsymbol{F}(t;\ \alpha, \beta)$ is the cumulative gamma distribution with parameters α, β. The parameters of cumulative gamma pdf, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ allow flexibility in the choice of trajectory to apply non-negativity constraints on the mitigation coefficients.

Parabolic Mitigation

The parabolic mitigation emission trajectory can be modeled (Socolow et al., 2007) as follows:

$$\boldsymbol{E(t) = E_{stab} + (E_o - E_{stab})}\,(1+ S\eta - (1+S)\,\eta^2) \qquad (6)$$

where $\boldsymbol{E(t)}$ = Emission at time t

$\boldsymbol{E_{stab}}$ = Emission target for stabilization year

$\eta = (t - t_o)/\tau_{PM}(t_o)$

t_o = current year

S = dimensionless parameter representing certain initial conditions of E(t) which results in a certain cumulative emission reduction.

$\tau_{PM}(t_o)$ = Time starting from t_o under parabolic mitigation trajectory

For any chosen stabilization target, the amount of additional atmospheric carbon content we can add to the atmosphere in the future is called the headroom, ***H(t)***.

Integrating (6) from $\eta = 0$ to 1,

$$\boldsymbol{H}(t_o) = [\boldsymbol{E}_{\mathbf{stab}} + (\boldsymbol{E}_{\mathbf{o}} - \boldsymbol{E}_{\mathbf{stab}})\,(S+4)/6]\,\tau_{PM}(t_o) \tag{7}$$

Equation (7) can be used to estimate S for a given headroom.

Mitigation Coefficients

We compute the mitigation responsibility function for a country or region by assuming a generalized linear responsibility function weighted by n variable factors:

$$\boldsymbol{R(t)} = \sum_{1}^{n} \lambda_i X_i \tag{8}$$

Where $\boldsymbol{\lambda_i}$ = i^{th} mitigation coefficient

$\boldsymbol{X_i}$ = i^{th} variable factor of Apportionment

Considering cumulative excess emissions and GDP as variable factors, the function will take the form:

Differentiated responsibility function, $\boldsymbol{R(t)}$=

$\lambda \times$ *Cumulative excess Emissions from a base Year* + $\mu \times$ GDP

$$\lambda, \mu > 0 \tag{9}$$

The difference between the actual or projected post-mitigation emissions and the entitled emissions constitute the excess emissions.

$\boldsymbol{E(t)}$ = Total CO_2 emissions in the year, t

$\boldsymbol{E_p(t)}$ = Total projected emissions in the year, t,(baseline)

$\boldsymbol{E_n(t)}$ = Entitled emissions in the year, t

$E(t) - E_n(t)$ = Excess emissions in the year, t

$E_p(t) - E(t)$ = Emission reduction in the year, t

$$E_p(t) - E(t) = \lambda\,(\mathbf{C} + \sum_{t_0}^{T-1}(E(t) - E_n(t)) + \mu \times \mathrm{GDP} \qquad (10)$$

where **C** = Cumulative excess emissions from the base year up to t_0. Emission entitlements, $E_n(t)$ are computed by calculating the per capita entitlement based on the targeted emissions by utilizing the principle of convergence. For example, if the target emission in 2030 is at current levels of 8.182 GtC, then the per capita entitled emissions would be 8.182/ Projected world population in the target stabilization year. This method makes the emission entitlements vary according to the set emission target.

On summation of (10) from $t = t_0$ to T,

$$\sum_{t_0}^{T}(E_p(t) - E(t)) = \lambda \times \sum_{t_0}^{T}[(\mathbf{C} + \sum_{t_0}^{T-1}(E(t) - E_n(t))] + \mu \times \sum_{t_0}^{T}\mathrm{GDP} \qquad (11)$$

Equations (10) and (11) yield the mitigation coefficients λ and μ.

Non-negativity constraints and apportionment

The non-negativity constraints on λ and μ are employed to determine the shape of the mitigation trajectory, both in parabolic and gamma mitigation. This is adjusted by modulating the values of the cumulative emission reduction during the period of mitigation. There is a window of feasible region of trajectories satisfying the non-negativity constraints which may be made use of to optimize the efficiency of mitigation. The following empirical equations are used to modulate the values of cumulative emission reduction for iterative convergence to satisfy the non-negativity constraints:

Cumulative Gamma pdf Mitigation:

$$(\text{Cumulative Emission Reduction})_{\text{new}} = (\text{Cumulative Emission Reduction})_{\text{old}} - \mathbf{0.5\,(\lambda / \mu^2)} \quad (12)$$

The rationale for this criterion is obvious from equation (11) which requires cumulative emission reduction to be positively correlated with -λ.

Parabolic Mitigation:

$$\mathbf{S_{new} = S_{old} + 0.5\,(\lambda/\mu^2)} \quad (13)$$

The rationale for this criterion is obvious from equation (7). As S is positively correlated with the head room, it has a negative correlation with the cumulative emission reduction.

Emission Reduction Flow Diagram

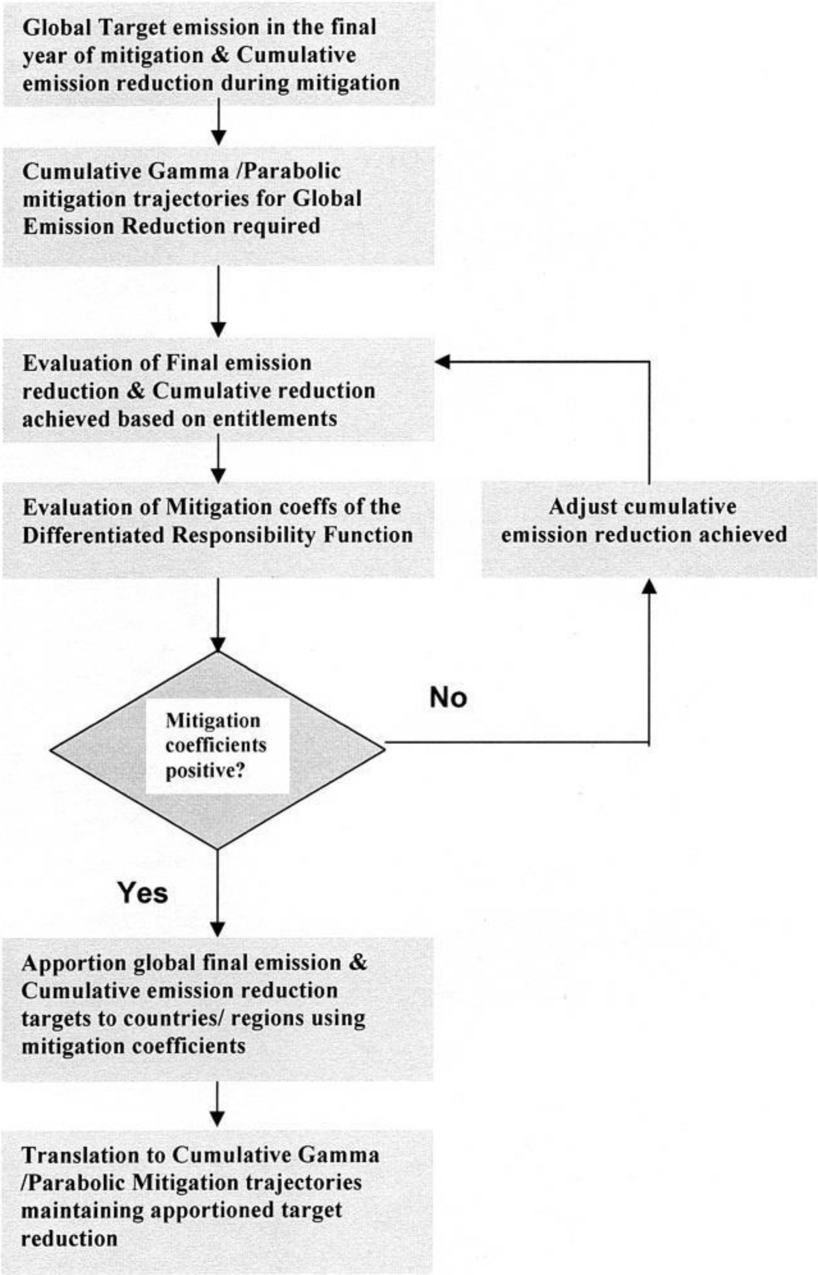
Apportionment of the global emission reduction targets are achieved through the global mitigation coefficients λ and μ. The mitigation targets so arrived at are translated to the corresponding emission trajectories, gamma mitigation or parabolic for the country or region.

The mitigation coefficient λ is the mapping parameter for cumulative excess emissions to the emission reduction responsibility function. It is a composite involving the relative contribution of cumulative excess emissions to the emission responsibility as well as the air-borne fraction of emissions that remain in the atmosphere. The mitigation coefficient μ is the mapping parameter for GDP to the emission reduction responsibility function. It is a composite involving the elasticity of emissions to GDP (which homogenizes the responsibility function) and the relative contribution of GDP to the emission responsibility. Considering the fact that population is a driving factor of emissions, population based entitlements have not been considered in the national GDP.

A suitable cumulative gamma fit for emission reduction during the mitigation period based on the targeted emissions in the final year is carried out taking into account the cumulative emission reduction required. This generates the required parameter values including the cumulative reduction during mitigation period which are used to evaluate the mitigation coefficients. The mitigation trajectory is then iteratively adjusted to satisfy the non-negativity constraints on the mitigation coefficients. These constraints project a window of possible range of mitigation trajectories and consequently a range of cumulative emissions during the mitigation period. We do not address the issue of optimization of these trajectories here.

The mitigation coefficients evaluated as above for global emissions can now be used to disaggregate and obtain differentiated responsibility functions of various countries/regions, based on the corresponding emission and income profiles. The mitigation targets so arrived at are again mapped to the corresponding cumulative gamma emission reduction paths for each country keeping the corresponding total reduction and the target reduction in the final year as translation constraints. This yields the emission reduction responsibilities and the corresponding trajectories for various countries/regions. The procedure is shown diagrammatically in Fig 6.

Figure 6: Evaluation Process Emission Reduction Trajectories of Countries/Regions



Only the emission levels at the target year and the cumulative emission reduction during the mitigation period are binding on the countries, the trajectory optimization can yield cost advantages by making use of the inherent strengths of various economies. The trajectories for various countries generated in this model are, therefore, representative and not conclusive. The relative stability of the binding commitment parameters with respect to different trajectories provides flexibility in optimization as well as uniformity of outcomes.